The Production Structure of Small, Medium-sized and Large Enterprises in Dutch Private Enterprise

Analysis at the aggregate level

Ton Kwaak

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Summary

Labour productivity differs significantly between small, medium-sized and large enterprises in the private enterprise sector. This holds both for the level of labour productivity as well as its development. Size-class differences regarding the level of labour productivity are only to a limited extent dependent on differences in sectoral structure. A model of the production structure has been developed and estimated to shed some light on these phenomena explaining various aspects of labour productivity of (average) small, medium-sized and large enterprises in the Dutch private enterprise sector. It should be stressed that the analysis refers to the 'representative' or average enterprise in the small, medium-sized or large enterprise segment of the economy; it does not refer to individual enterprises.

Labour productivity in 2009
Labour productivity differentials between small, medium-sized and large enterprises can be explained by the following factors:
- The shape of the production function. This refers in particular to the existence of fixed labour, and the marginal productivity of labour and capital varying across size-classes.
- Differences in capital use. As a result of differences in the shape of the production function and differences in labour costs relative to capital costs across size-classes, the capital intensity of production is much higher in large enterprises compared to SMEs. This significantly contributes to labour productivity in large enterprises.

If labour productivity were calculated assuming the same capital/labour ratio across all size-classes – which is not optimal! – then labour productivity in SMEs would in fact be higher than in large enterprises. So observed differences in labour productivity are the result of differences in the shape of the production function and – within the conditions set by the production function – of entrepreneurial choices. From this point of view the relatively low labour productivity of SMEs is no issue for direct concern.

Labour productivity growth
Regarding differences in labour productivity growth in small, medium-sized and large enterprises, the following factors have played a major role in the structural development labour productivity between 1993 and 2009:
- Labour-augmenting technical progress. In conformity with other studies, technical progress is assumed to be labour-augmenting. The impact of labour-augmenting technical progress is by far highest in small enterprises, and below average in medium-sized and large enterprises.
- Capital/labour substitution. Variable labour is substituted by capital if labour costs increase more than capital costs, and vice versa. This effect has been rather weak in medium-sized enterprises, and slightly above average in small and large enterprises.
- Changes in the share of fixed labour. This has negatively affected labour productivity growth in all size-classes, but mostly so in small enterprises because of the high share of fixed labour in this size-class.
The 2009 Great Recession has negatively affected labour productivity in the private enterprise sector. This has mainly been the result of decreasing capacity utilisation, and of labour hoarding. The latter only occurred in medium-sized and large enterprises; small enterprises in fact closed the gap between labour demand and actual employment. On the other hand, due to shrinking enterprise size, the share of fixed labour in total employment in small enterprises increased significantly, which in turn contributed negatively to labour productivity growth in small enterprises. This is, however, not strictly related to the Great Recession, but fits well in the long-term trend of decreasing enterprise size.

Further research
The model described here might be extended in two directions. First, it may be applied to individual economic sectors. This is dealt with in a separate paper (Kwaak, 2012). Secondly, following the assumption of full competition in markets, the model has implications for the number of enterprises which have not yet been reviewed. Finally, a more thorough statistical substantiation of the current analysis may be obtained by analysing the short run development of employment and prices. This can be done by assuming that there is a long run tendency for actual employment and actual prices to move towards equilibrium levels (similar to an Error Correction framework).
1 Introduction

In 2009, the Dutch private enterprise sector\(^1\) provides 4.7 million jobs. 60% of these are in small and medium-sized enterprises (SMEs)\(^2\), with the remainder in large enterprises (LSEs). The share of SMEs in value added is 45%, which is significantly less than their employment share. This signifies a below average labour productivity: 54,000 €/labour year in SMEs, against 97,000 €/labour year in large enterprises.

Table 1 size and structure of private enterprise, 1993 and 2009

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<td>mn. €</td>
<td>1.000 €/labour year</td>
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<tr>
<td>small enterprises</td>
<td>378.000</td>
<td>640.000</td>
<td>1.115.000</td>
<td>1.323.000</td>
<td>31.000</td>
<td>55.000</td>
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<td>41</td>
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<tr>
<td>medium-sized</td>
<td>40.000</td>
<td>57.000</td>
<td>1.151.000</td>
<td>1.466.000</td>
<td>50.000</td>
<td>95.000</td>
<td>43</td>
<td>65</td>
</tr>
<tr>
<td>small and medium-</td>
<td>418.000</td>
<td>697.000</td>
<td>2.266.000</td>
<td>2.789.000</td>
<td>81.000</td>
<td>149.000</td>
<td>36</td>
<td>54</td>
</tr>
<tr>
<td>large enterprises</td>
<td>4.000</td>
<td>5.000</td>
<td>1.670.000</td>
<td>1.870.000</td>
<td>82.000</td>
<td>182.000</td>
<td>49</td>
<td>97</td>
</tr>
<tr>
<td>all enterprises</td>
<td>422.000</td>
<td>702.000</td>
<td>3.936.000</td>
<td>4.659.000</td>
<td>163.000</td>
<td>331.000</td>
<td>41</td>
<td>71</td>
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* Employment in labour years; including self-employed.
** Gross value added at factor costs.
*** Value added/employment.
Source: Panteia/EIM.

Over the last 15 years, real value added has increased faster than employment, indicating increasing labour productivity. Employment growth has been strongest in SMEs, while real value added growth has been highest in LSEs. Thus, the labour productivity gap between SMEs and LSEs has widened\(^3\).

To some extent, labour productivity differences between enterprise size-classes can be attributed to differences in sectoral structure. For instance, many SMEs are active in trade and the hospitality sector, which has a low average labour productivity and which also has experienced low productivity growth. However, differences in sectoral structure explain only a small part of observed productivity differences (Kwaak, 2011). This calls for a more in-depth analysis of the production structure of the Dutch private enterprise sector by enterprise size-class. Elements of such an analysis should be:

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1 Private enterprise refers to all enterprises excluding agriculture, mining and quarrying, public utilities, real estate, and health care. In terms of the NACE Rev. 2 classification of economic activities: sections C, F –K, M, N, R, S.

2 The traditional Dutch SME-definition is used throughout. Small enterprises are enterprises having less than 10 occupied persons, medium-sized enterprises have between 10 and 100 occupied persons, and large enterprises have over 100 occupied persons.

3 For a similar conclusion at EU-level, see Kwaak, Wennekers and Van Stel (2010).
− Capital intensity: differences in capital intensity could be a source of labour productivity differentials;
− Role of fixed costs, that could be relatively large in small enterprises;
− Capital/labour substitution: variations in the extent to which labour has been substituted by capital could explain variations in labour productivity growth;
− Technical progress that could vary with enterprise size.
These are all elements that are related to the production function. To put such an analysis in a wider perspective (with the additional advantage of including more variables and observations in the empirical analysis), the analysis is extended to include factor demand and price setting as well.
The current paper presents such a model\(^1\). It starts by assuming a production function for value added at the enterprise level with capital and *variable* labour as its arguments. There are fixed costs that are associated with labour. There is labour augmenting technical progress. Then, assuming enterprises behave as profit maximisers, factor demand equations are derived. Furthermore, assuming no excess profits in the long run, an equilibrium price equation is added to the model; this allows for a broader testing of the model. This theoretical structure is described in Chapter 2. Parameter values are established for individual enterprise size-classes. Estimation methodology and results are presented in Chapters 3 and 4. Then, Chapter 5 analyses the labour productivity differentials between small, medium-sized and large enterprises in the private enterprise sector using the model. Chapter 6 discusses the overall results.
The current paper focuses on results at the level of the private enterprise sector, with special emphasis on methodological issues. In a corresponding paper results by economic sector will be presented.

\(^1\) The model is an extension of the one presented in Kwaak (2011). Main differences are the inclusion of a price equation, and the treatment of fixed costs. Furthermore, the estimation method has been refined.
2 Theoretical model

There are three key elements in the model: the production function, profit maximising behaviour of individual enterprises, and full competition. The production function (section 2.1) describes how enterprises combine capital and labour to produce value added. Using the profit maximising assumption, factor demand equations are derived (section 2.2). The full competition assumption allows deriving price equations which are described in section 2.3. Equations for the production function and factor demand are formulated at the level of the representative enterprise; results at the aggregate level are obtained by aggregation over enterprises. The price equation builds on the assumption of full competition (and therefore, no excess profits) at the aggregate level.

2.1 Production function

2.1.1 At the enterprise level

Firms produce value added\(^1\) according to a production function in variable labour and capital. Thus, there are fixed costs that completely consist of labour cost, i.e., at the firm level a certain amount of labour is required to keep the firm going\(^2\). Using \(l\) for total employment in the individual enterprise\(^3\), \(l_f\) for fixed labour, and \(l_v\) for variable labour, the following definition applies:

\[ l_v = l - l_f \]

where

- \(l\) total employment in the enterprise (in physical units)
- \(l_f\) fixed labour in the enterprise (in physical units; \(l_f < l\))
- \(l_v\) variable labour in the enterprise (in physical units)

Technical progress is variable labour augmenting, at an annual rate of \(\lambda\)\(^4\). Therefore, with \(\tau\) an arbitrarily chosen base-year, variable labour in efficiency units equals:

\[ l_{ve} = l_v \cdot (1 + \lambda)^{(t-\tau)} \]

where

- \(l_{ve}\) variable labour in the enterprise (in efficiency units)
- \(l_v\) variable labour in the enterprise (in physical units)
- \(\lambda\) rate of labour-augmenting technical progress
- \(t\) time (\(\tau\) being an arbitrarily fixed base-year)

\(^1\) Gross value added at factor costs, in short 'production'.

\(^2\) Even though fixed labour does not contribute directly to production, its existence is a \textit{sine qua non} for the enterprise's existence, and hence its production.

\(^3\) Lowercase letters indicate variables at the enterprise level. Uppercase letters denote aggregated values, calculated by multiplying firm level values by the number of enterprises \(N\).

\(^4\) Capital augmenting technical progress is not relevant for The Netherlands (Lunsing, 2011-a).
The net change of capital stock equals the balance of gross investment and depreciation. The depreciation rate of capital is $\delta$. Therefore:

\begin{align}
(3) & \quad k_t = k_{t-1} - d + j \\
(4) & \quad d = k_{t-1} \cdot \delta
\end{align}

where
- $k$ capital stock in the enterprise
- $d$ depreciation in the enterprise
- $j$ gross investment in the enterprise
- $\delta$ depreciation rate

The production function is CES in capital and variable labour:

\[ y = \beta \cdot \left\{ \alpha \cdot k - \rho + (1-\alpha) \cdot l_{ve} \right\}^{1/\rho} \]

where
- $y$ gross value added at factor cost in the enterprise
- $k$ capital stock in the enterprise
- $l_{ve}$ variable labour in the enterprise (in efficiency units)
- $\beta$ scaling constant ($\beta > 0$)
- $\alpha$ distribution constant ($0 < \alpha < 1$)
- $\rho$ substitution parameter ($\rho > -1$ and $\rho \neq 0$); the substitution elasticity between capital and variable labour $\sigma = 1/(1 + \rho)$

### 2.1.2 At the aggregate level

Assuming all enterprises within an enterprise size-class to be identical, aggregate values are obtained by multiplying variables at the enterprise level by the number of enterprises.

For labour, the model at the aggregate level reads as:

\begin{align}
(6) & \quad L = N \cdot L_f \\
(7) & \quad L_v = L - L_f
\end{align}

where
- $L$ total aggregate employment (in physical units)
- $L_f$ total variable labour (in physical units)
- $L_v$ total variable labour (in physical units)
- $N$ number of enterprises

Aggregate labour use in efficiency units is:

\[ L_{ve} = L_v \cdot (1 + \lambda)^{(t-1)} \]

where
- $L_{ve}$ aggregate variable labour (in efficiency units)
- $\lambda$ rate of labour-augmenting technical progress
- $t$ time ($\tau$ being an arbitrarily fixed base-year)
For aggregate capital the laws of motion are:

\[ K_t = K_{t-1} - D + J \]  
\[ D = K_{t-1} \cdot \delta \]

where
- \( K \) aggregate capital stock
- \( D \) aggregate depreciation
- \( J \) aggregate gross investment
- \( \delta \) depreciation rate

Finally, the aggregate production function is:

\[ Y = \beta \cdot \left\{ \alpha \cdot K^\rho + (1-\alpha) \cdot L_{ve}^\rho \right\}^{1/\rho} \]

where
- \( Y \) aggregate gross value added at factor cost
- \( K \) aggregate capital stock
- \( L_{ve} \) aggregate variable labour (in efficiency units)
- \( \beta \) scaling constant (\( \beta > 0 \))
- \( \alpha \) distribution constant (0 < \( \alpha < 1 \))
- \( \rho \) substitution parameter; note that the substitution elasticity between capital and variable labour \( \sigma = 1/(1 + \rho) \) (\( \rho > -1 \), \( \rho \neq 0 \))

### 2.2 Factor demand

#### 2.2.1 At the enterprise level

Profits are \( \pi = y \cdot p_y - k \cdot p_k - l \cdot p_l \). Assuming wages do not differ between variable and fixed labour, substituting (1), (2) and (5) gives profits as a function of factor use, factor prices and output price (which are exogenous to the enterprise):

\[ \pi = \beta \cdot \left\{ \alpha \cdot K^\rho + (1-\alpha) \cdot L_{ve}^\rho \right\}^{1/\rho} \cdot p_y - k \cdot p_k - (L_{ve}/(1+\lambda)^{(t-1)} + l) \cdot p_l \]

It is assumed that entrepreneurs seek to maximise profits. From the first order condition \( \partial \pi / \partial k = 0 \) and substituting back (5), optimal capital use can be calculated as:

\[ k^* = \frac{\beta \cdot \sigma \cdot \alpha \cdot y \cdot (p_k/p_y)^\sigma}{} \]

where
- \( k^* \) optimal capital use in the enterprise
- \( y \) production
- \( p_k \) user cost of capital
- \( p_y \) output price

Optimal capital use depends on production as well as the relative costs of capital.

Similarly, setting \( \partial \pi / \partial L_{ve} = 0 \), optimal labour use in efficiency units is

\[ L_{ve}^* = \beta \cdot \sigma \cdot (1-\alpha)^\sigma \cdot y \cdot \left\{ [p_l/(1 + (1+\lambda)^{(t-1)})]/p_y \right\}^\sigma \]

Using this and (12), the optimal capital/variable labour ratio equals:

\[ k^*/L_{ve}^* = \left[ \alpha/(1-\alpha)^\sigma \cdot (p_k/[p_l/(1 + (1+\lambda)^{(t-1)})]) \right]^\sigma \]
Optimal variable labour use in efficiency units equals capital stock, divided by the optimal capital/variable labour ratio \( k^*/l^* \). Using (2) to calculate optimal labour use in physical units, and (1) to take into account fixed labour, total labour demand in the enterprise equals:

\[
(13) \quad l^* = l_f + k \cdot \left[ \alpha/(1-\alpha) \right]^{\sigma} \cdot \left\{ \frac{pl/(1 + \lambda)^{t-\tau}}{pk} \right\}^{\sigma} \cdot \left\{ \frac{pl/(1 + \lambda)^{t-\tau}}{pk} \right\}^{\sigma} / \left[ (1 + \lambda)^{(t-\tau)} \right]
\]

where

- \( l^* \): optimal total labour use in the enterprise (labour demand)
- \( k \): capital stock in the enterprise
- \( l_f \): fixed labour in the enterprise
- \( \alpha \): distribution constant (0 < \( \alpha \) < 1)
- \( \sigma \): substitution elasticity
- \( \lambda \): rate of labour-augmenting technical progress
- \( pk \): user cost of capital
- \( pl \): wage rate

Labour demand depends on the amount of capital, and the cost of labour relative to capital. Additionally, there is an amount of fixed labour. In statistics, enterprise size is often measured in terms of employment. Optimal enterprise size \( l^* \) is seen to depend here on fixed labour \( (l_f; \) treated as a parameter here, but see section 2.4), as well as capital and relative labour costs. It is therefore not constant, which is consistent with observed changes in average enterprise size.

\( pk \) is the user costs of capital, which is equal to the price of capital goods, adjusted for interest, depreciation and a mark-up for entrepreneurial risk:

\[
(14) \quad pk = pj \cdot (r + \delta + \theta)
\]

where

- \( pk \): user cost of capital
- \( pj \): price of capital goods
- \( r \): interest rate
- \( \delta \): depreciation rate
- \( \theta \): mark-up for entrepreneurial risk

### 2.2.2 At the aggregate level

It was assumed that all enterprises within an enterprise size-class are identical. Furthermore, factor and output prices are equal for all enterprises within a size-class. Therefore, aggregate quantities are again obtained by multiplying variables at the enterprise level by the number of enterprises, \( N \).

So aggregate demand for capital is:

\[
(15) \quad K^* = \beta^{\rho e} \cdot \alpha^e \cdot \gamma (pk/py)^{\sigma}
\]

where

- \( K^* \): optimal capital stock at the aggregate level
- \( \gamma \): aggregate production
- \( pk \): user cost of capital
- \( py \): output price
- \( \beta \): scaling constant (\( \beta > 0 \))
- \( \alpha \): distribution constant (0 < \( \alpha \) < 1)
- \( \sigma \): substitution elasticity
The aggregate labour demand function is:

\[ L^* = N \cdot l_f + K \cdot \frac{[\alpha/(1-\alpha)]^{[\alpha/(1+\alpha)]}}{[\alpha/(1+\lambda)^{(1-\alpha)}]} \cdot \frac{pl/(1 + \lambda)^{(1-\sigma)} / \{pl + [1/(1 + \lambda)^{(1-\sigma)}] \}}{[1 + \lambda)^{(1-\sigma)}]} \]

where

- \( L^* \) total aggregate labour demand (in physical units)
- \( K \) aggregate capital stock
- \( N \) number of enterprises
- \( l_f \) fixed labour in the enterprise
- \( \beta \) scaling constant (\( \beta > 0 \))
- \( \alpha \) distribution constant (0 < \( \alpha < 1 \))
- \( \sigma \) substitution elasticity
- \( \lambda \) rate of labour-augmenting technical progress
- \( pk \) user cost of capital
- \( pl \) wage rate

### 2.3 Equilibrium prices

Under full competition, prices are such that no excess profits occur, i.e., in equilibrium profits \( \pi = y \cdot py - k \cdot pk - l \cdot pl \) (section 2.2) are zero. Fixed costs are a mark-up over variable costs. Excluding fixed costs, using (1), (2), (12) and (13), the optimal price equals:

\[ py^* = \beta^{-1} \cdot \{\alpha \cdot pk^{1-\sigma} + (1-\alpha) \cdot pl/(1 + \lambda)^{(1-\sigma)} \} \]

With \( w_f \) denoting the share of fixed costs in total costs (\( w_f = N \cdot l_f \cdot pl/Y \cdot py \), which is not constant\(^2\), the equilibrium output price is \( py^* = w_f \cdot pl + (1 - w_f) \cdot py^* \), so:

\[ py^* = w_f \cdot pl + (1 - w_f) \cdot \beta^{-1} \cdot \{\alpha \cdot pk^{1-\sigma} + (1-\alpha) \cdot pl/(1 + \lambda)^{(1-\sigma)} \} \]

where

- \( py^* \) equilibrium output price
- \( w_f \) share of fixed costs in total production
- \( \beta \) scaling constant (\( \beta > 0 \))
- \( \alpha \) distribution constant (0 < \( \alpha < 1 \))
- \( \sigma \) substitution elasticity
- \( \lambda \) rate of labour-augmenting technical progress
- \( pk \) user cost of capital
- \( pl \) wage rate

### 2.4 Fixed labour costs revisited

The role of fixed labour was explained in section 2.1. Parameter \( l_f \) – the amount of fixed labour per enterprise – was implicitly viewed as constant. However, there is no a priori reason to believe that fixed labour is constant: fixed labour is labour that does not contribute directly to production. It seems fair to assume that to some extent the amount of fixed labour per enterprises varies with enterprise size. It does vary across size-classes (Kwaak, 2011), and a comparison of fixed costs across size-classes suggests economies of scale: the share of fixed labour in total labour use is largest in small enterprises, and smallest in

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1 The user cost of capital \( pk \) already include a ‘normal’ entrepreneurial remuneration.

2 By assumption, wages do not differ between fixed and variable labour.
large enterprises. Over time, fixed labour may also vary with average enterprise size within enterprise size-classes. In view of this, an alternative version of the production model from section 2.1 is reviewed in which the following equation is used:

\[
(18) \quad l_f = \phi_1 + \phi_2 \cdot \ln(l)
\]

where

\( l_f \) fixed labour at the enterprise level  
\( l \) total employment in the enterprise (as a proxy for enterprise size)

With \( \phi_2 = 0 \), the model is equivalent with constant fixed labour per enterprise \( l_f = \phi_1 \), implying \( \phi_1 \) to be non-negative. With \( \phi_1 = 0 \), fixed labour would vary with labour; economies of scale suggest \( 0 < \phi_2 < 1 \).
3 Estimation strategy

3.1 Basic methodology

For each enterprise size-class, values for the following model-parameters should be established:

- \( \beta \) scaling constant (\( \beta > 0 \))
- \( \alpha \) distribution constant (\( 0 < \alpha < 1 \))
- \( \rho \) substitution parameter (\( \rho > -1 \) and \( \rho \neq 0 \)); the substitution elasticity between capital and variable labour \( \sigma = 1/(1 + \rho) \)
- \( \delta \) depreciation rate (%)
- \( \theta \) mark-up for entrepreneurial risk (%)
- \( \lambda \) rate of labour-augmenting technical progress
- \( \alpha_1, \alpha_2 \) parameters relating fixed labour per enterprise to enterprise size

Additionally, initial values for the capital stock (i.e., the capital stock prior to the estimation period) must be estimated, as no capital stock data per enterprise size-class exist\(^1\).

There is information at the aggregate level (i.e., the aggregate of small, medium-sized and large enterprises) that should be taken into account:

- From Lunsing (2011-a) an (average) value of \( \theta = 2 \) could be derived, which should hold across all sectors and size-classes\(^2\). Also aggregate values for \( \lambda \) and \( \sigma_{\text{tot}} \) could be derived. Here \( \sigma_{\text{tot}} \) is the overall substitution elasticity, taking into account the fact that only variable labour reacts to changes in relative labour costs. For elementary size-classes, it is calculated as \( \sigma_{\text{tot}} = \frac{(1 - L_f/L)}{\sigma} \).
- Aggregates across size-classes are calculated using production \( Y \) as weights. The average value for \( \lambda \) amounts to 2.1, the average value for \( \sigma_{\text{tot}} \) equals 0.5.
- Smid and Vromans (2006) provide information on the share of fixed labour in total employment (\( L_f/L \)), which is equal to 20% on average across enterprise size-classes.
- Statistics Netherlands provides information on total capital stock\(^3\).

Parameters should be fitted on available time-series on real gross value added at factor costs (\( Y \)), depreciation (\( D \)), employment (\( L \)), and the deflator of gross value added at factor costs (\( py \)). For each variable \( w \), an estimated value \( \hat{w} \) can be calculated, using observed values of variables and estimated parameters. A goodness-of-fit statistic is \( \text{PSR}^2 = 1 - \frac{\sum (w - \hat{w})^2}{n}/\text{var}(z) \), with var the variance operator on the time-series, and \( n \) the length of the time-series. This variable is closely related to the \( R^2 \) statistic from standard regression techniques\(^4\).

\(^1\) An important additional result of this study is that estimated capital stock series by enterprise size-class are obtained that were not available before.

\(^2\) If \( \theta \) were not equal across size-classes (or sectors of industry), capital would flow from segments with low \( \theta \) to segments with high \( \theta \) (unless risk varies systematically across size-classes).

\(^3\) See Van den Bergen, De Haan, De Heij and Horsten (2005), and Van den Bergen, De Haan, De Heij and Horsten (2009).

\(^4\) An important difference is the following. In the simple linear regression model \( z = a_0 + a_1 \cdot x \), the average residual \( \sum (z - \hat{z})/n \) is zero. In the current application this need not be the case.
The estimator used maximises the sum of the PSR2’s across all variables\(^1\) and size-classes by choosing values for the parameters, taking into account the cross-size-class restrictions described above (see Annex I for a formal representation). This least-squares estimator is highly non-linear in the parameters, and has no analytic solution. Instead, numerical optimisation is used\(^2\).

3.2 Obtaining confidence levels

The estimator used does not provide standard errors for parameters analytically. Instead, bootstrapping (introduced by Efron, 1979) is used to estimate confidence intervals. Pseudo-values for the parameter values are produced, and the variability of these pseudo-estimates is assumed to represent the standard errors of the estimated parameters. The procedure used (following Ziari and Azzam, 1999) is as follows:

- Randomly re-order the residual series \(e_{i,t}\) from the original estimate for each variable \(i\) (\(i= Y, D, L, py\)). The re-ordered series are denoted \(v_{i,t}\);
- Calculate pseudo-observations \(q^\ast_{i,t} = q_{i,t} - v_{i,t}\), with \(q_{i,t}\) the observed value of variable \(i\) in year \(t\);
- Apply the estimator described above to the pseudo-observations obtained. This procedure is repeated many (here, 200) times, giving a fair impression of the variation in each parameter.

One way to calculate confidence intervals from the bootstrapping estimates is via their variance\(^3\). Efron and Tibshirani (1993) demonstrate that (under some conditions) bootstrapping is asymptotically consistent. An estimate of the t-value could thus be obtained by dividing the estimated parameter by the standard deviation of the bootstraps. On the other hand, bootstrapping may be overly optimistic, in particular because in this specific case, time series are rather short. Also, the distribution of some parameters is asymmetric, such as the substitution parameter \(\rho\) (forced to be strictly positive). Alternatively, a non-parametric way to obtain confidence intervals is the percentile method. In case of 200 replications one would take the 6\(^{th}\) lowest and the 6\(^{th}\) highest as the lower and upper bound of the 95% confidence interval (Ziari and Azzam, 1999). Both approaches are followed here.

3.3 Data

The data have been constructed in the framework of the research program ‘SMEs and Entrepreneurship’ implemented by Panteia/EIM and financed by the Ministry of Economic Affairs, Agriculture and Innovation\(^4\). They are based on several sources, especially the ‘Production Statistics’ and the National Accounts published by Statistics Netherlands. Data are fully consistent with national Accounts. The basic methodology is described in Kwaak and Liebregts (2012).

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\(^1\) The rationale for this is that the PSR2’s are dimensionless numbers and do not depend on the unit of measurement.

\(^2\) A generalised reduced gradient algorithm is used; see for instance Lasdon and Smith, 1992.

\(^3\) Hall (1988) and Dicico and Romano (1988) provide an overview of methods for obtaining confidence intervals from the bootstrap.

\(^4\) See http://www.entrepreneurship-sme.eu/.
4 Estimation results

First, the full model established in Chapter 2 has been estimated using the methodology set out in Chapter 3; these results are presented in section 4.1. The initial parameter estimates have been checked on their significance using the bootstrapping procedure set out in section 3.2. Also, size-class differences between parameters are reviewed. Furthermore, as it turned out that the originally established relation between fixed labour and enterprise size is not significant, an alternative formulation has been estimated. This is discussed in section 4.2. Section 4.3 analyses actual production and production capacity, as well as equilibrium and actual labour employment and prices, which can be calculated by simulating the model obtained.

4.1 Initial estimates

Point estimates and significance of parameters

Initially a free estimate of all parameters from the model from the previous chapter has been obtained. Table 2 presents these initial results, including measures for confidence levels as discussed in section 3.2. Averages across size-classes are added.

The statistical significance of $\beta$ is of minor relevance here. Regarding $\alpha$, both $\alpha$ and $(1 - \alpha)$ should significantly differ from 0, which is the case here. Note that the magnitude (but not the significance) of $\alpha$ also depends amongst other things on the units of measurement chosen.

The average amount of fixed labour, $l_f$, is significantly different from zero for all three size-classes, whereas the parameter that relates fixed labour to enterprise size, $\alpha_2$, is not. Apparently, the variation over time in average enterprise size within size-classes has been too small to make inferences on $\alpha_2$. The substitution parameter $\rho$ is not significantly different from zero in small and medium-sized enterprises; for large enterprises, the estimate is significantly positive. Conversely, the $\lambda$ parameter of labour augmenting technical progress is significantly positive for small enterprises while it is not for medium-sized and large enterprises. All estimated parameters on the capital stock (i.e., the initial capital stock $K_{1992}$ and the depreciation rate $\delta$) are significantly different from zero.

---

1 $\alpha$ is bounded between 0 and 1. With $\alpha$ not significant from 0, there would be no significant impact of capital on production capacity; with $(1 - \alpha)$ not significant from 0, there would be no significant impact of labour on production capacity.

2 For stability reasons (at $\rho = 0$, the production function changes into a Cobb-Douglas form), it has been bounded from below at a value of 0.001.
<table>
<thead>
<tr>
<th>Table 2</th>
<th>initial parameter estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>small enterprises</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.62</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.63</td>
</tr>
<tr>
<td>(6/194)</td>
<td>(0.61/0.71)</td>
</tr>
<tr>
<td>(st. dev.)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>$\varphi_1$</td>
<td>1.38</td>
</tr>
<tr>
<td>(6/194)</td>
<td>(1/1.69)</td>
</tr>
<tr>
<td>(st. dev.)</td>
<td>(0.24)</td>
</tr>
<tr>
<td>$\varphi_2$</td>
<td>0.08</td>
</tr>
<tr>
<td>(6/194)</td>
<td>(0.06/1)</td>
</tr>
<tr>
<td>(st. dev.)</td>
<td>(0.32)</td>
</tr>
<tr>
<td>average $l_i$</td>
<td>lab. yrs. / enterprise</td>
</tr>
<tr>
<td>(6/194)</td>
<td>(1.45/1.52)</td>
</tr>
<tr>
<td>(st. dev.)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>average $L_i/L$</td>
<td>(in %)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.00</td>
</tr>
<tr>
<td>(6/194)</td>
<td>(0/0)</td>
</tr>
<tr>
<td>(st. dev.)</td>
<td>(0)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.00</td>
</tr>
<tr>
<td>$\sigma_{tot}$</td>
<td>0.44</td>
</tr>
<tr>
<td>(6/194)</td>
<td>(0.42/0.45)</td>
</tr>
<tr>
<td>(st. dev.)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>(in %)</td>
</tr>
<tr>
<td>(6/194)</td>
<td>(5.04/6.94)</td>
</tr>
<tr>
<td>(st. dev.)</td>
<td>(0.54)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>(in %)</td>
</tr>
<tr>
<td>(6/194)</td>
<td>(7.07/13.64)</td>
</tr>
<tr>
<td>(st. dev.)</td>
<td>(1.55)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>(in %; fixed)</td>
</tr>
<tr>
<td>million 2004</td>
<td>Euro</td>
</tr>
<tr>
<td>$K_{1992}$</td>
<td>(15,265/66,484)</td>
</tr>
<tr>
<td>(st. dev.)</td>
<td>(7.232)</td>
</tr>
<tr>
<td>2001 capital stock</td>
<td>million Euro</td>
</tr>
</tbody>
</table>

st. dev.: standard deviation of 200 bootstrap estimates.
Source: Panteia/EIM.
Differences between small, medium-sized and large enterprises

Table 3 provides an analysis of the size-class difference for selected parameters: the substitution parameter $\rho$ and the overall substitution parameter $\sigma_{\text{tot}}$, the rate of labour-augmenting technical change $\lambda$, average fixed labour $l_f$, the depreciation rate $\delta$ and the initial capital stock $K_{1992}$. The table shows the differences between the point estimates from Table 2, as well as the estimated standard deviation of these differences.\(^1\)

Regarding the substitution parameter $\rho$ (and therefore the substitution elasticity between capital labour and variable labour, $\sigma$), difference between small and medium-sized enterprises are negligible, whereas the parameter for large enterprises is significantly higher. The overall substitution parameter $\sigma_{\text{tot}}$ is significantly higher in medium-sized enterprises than in small and large ones.\(^2\)

Differences regarding labour-augmenting technical change ($\lambda$) mainly concern the high value in small enterprises vis-à-vis the low values for medium-sized and large ones which are significant. Differences between medium-sized and large enterprises are not significant.

Size-class differences regarding the amount of fixed labour ($l_f$) conform the hypothesis that they are positively correlated with enterprise size (also compare equation (18), section 2.4). On average (note that parameter $\alpha_2$ is not significant different from zero), in both medium-sized and large enterprises, they are significantly higher than in small ones, the difference between medium-sized and large enterprises not being significant.

Size-class differences regarding the 1992 capital stock ($K_{1992}$) clearly are significant. Perhaps more remarkable is the result that depreciation rate $\delta$ is negatively correlated with enterprise size. In particular for small enterprises, the value found is significantly higher than for medium-sized and large enterprises. This may have to do with a different structure of capital goods, for instance fewer buildings and structures (which have a low depreciation rate) in small enterprises' capital stock; however, there is no information on this subject.\(^3\)

---

\(^1\) These standard deviations are based on the 200 bootstrap estimates. The standard deviation of the difference $\Delta$ between two parameters $A$ and $B$, $\sigma_{\Delta}$, has been calculated as $\sigma_{\Delta}^2 = \sigma_A^2 + \sigma_B^2 - 2 \cdot \text{covariance}(A, B)$. The latter correction is needed as information on the average value across size-classes has been used as a restriction on size-class parameters which therefore tend to be negatively correlated.

\(^2\) As indicated above, the overall substitution parameter $\sigma_{\text{tot}}$ is calculated by multiplying the substitution elasticity between capital labour and variable labour ($\sigma$) with the share of variable labour in total employment ($L_v/L$). This is because fixed labour is not subject to capital/labour substitution.

\(^3\) It may be the case that small enterprises tend to rent (in stead of own) buildings and infrastructures, due to amongst others financing issues.
**Table 3**  size-class differences between selected initial parameter estimates*

<table>
<thead>
<tr>
<th>differences between size-classes</th>
<th>$\rho$</th>
<th>differences between size-classes</th>
<th>$\sigma_{\text{tot}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$mb$</td>
<td>$gb$</td>
<td>$mb$</td>
</tr>
<tr>
<td>difference to $kb$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- difference</td>
<td>0.01</td>
<td><strong>2.28</strong></td>
<td></td>
</tr>
<tr>
<td>- st. dev</td>
<td>0.45</td>
<td>0.48</td>
<td></td>
</tr>
<tr>
<td>difference to $mb$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- difference</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- st. dev</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>differences between size-classes</td>
<td>$\lambda$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$mb$</td>
<td>$gb$</td>
<td>$mb$</td>
</tr>
<tr>
<td>difference to $kb$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- difference</td>
<td><strong>-5.18</strong></td>
<td><strong>-4.58</strong></td>
<td></td>
</tr>
<tr>
<td>- st. dev</td>
<td>0.59</td>
<td>0.93</td>
<td></td>
</tr>
<tr>
<td>difference to $mb$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- difference</td>
<td>0.59</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- st. dev</td>
<td>0.55</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>differences between size-classes</td>
<td>$K_{1992}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$mb$</td>
<td>$gb$</td>
<td>$mb$</td>
</tr>
<tr>
<td>difference to $kb$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- difference</td>
<td>11,047</td>
<td><strong>187,073</strong></td>
<td></td>
</tr>
<tr>
<td>- st. dev</td>
<td>26,918</td>
<td>36,064</td>
<td></td>
</tr>
<tr>
<td>difference to $mb$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- difference</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- st. dev</td>
<td>47,191</td>
<td>176,026</td>
<td></td>
</tr>
</tbody>
</table>

* Difference between point estimates, standard deviation using the 200 bootstrap estimates. Boldface indicates more than twice the standard deviation. Source: Panteia/EIM.

**4.2 Selected model**

The previous analysis suggests that coefficient $\alpha_2$ from equation (18) (section 2.4) cannot be estimated effectively by individual size-class due to insufficient variation over time in the average enterprise size ($l$) within individual size-
classes. In stead, it is assumed that $\varphi_1$ and $\varphi_2$ do not vary across size-classes. This utilises the large differences in average enterprise size between size-classes to establish the relation between fixed labour and enterprise size. The other parameters, even though they are not significantly different from zero in some cases, have been estimated freely. In particular, assuming technical progress to be zero in medium-sized and large enterprises seems at odds with common knowledge, and their low values in the initial estimate are at least partly the result of the restriction on the value of the average rate of technical progress. Estimation results do differ only slightly from the free initial estimates; they are presented in Table 4.

Table 4 parameter estimates selected model

<table>
<thead>
<tr>
<th></th>
<th>small enterprises</th>
<th>medium-sized enterprises</th>
<th>large enterprises</th>
<th>aggregate small, medium-sized and large enterprises</th>
<th>benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.68</td>
<td>0.37</td>
<td>0.17</td>
<td>0.32</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.65</td>
<td>0.56</td>
<td>0.06</td>
<td>0.31</td>
<td></td>
</tr>
<tr>
<td>$\varphi_1$</td>
<td></td>
<td></td>
<td></td>
<td>1.38 *</td>
<td></td>
</tr>
<tr>
<td>$\varphi_2$</td>
<td></td>
<td></td>
<td></td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>average $l_f$</td>
<td>lab.yrs. / enterprise</td>
<td>1.59</td>
<td>2.08</td>
<td>2.66</td>
<td>1.64</td>
</tr>
<tr>
<td>average $L_f/L$</td>
<td>(in %)</td>
<td>60</td>
<td>8</td>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.00</td>
<td>0.01</td>
<td>2.20</td>
<td>1.17</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.00</td>
<td>0.99</td>
<td>0.31</td>
<td>0.63</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{tot}$</td>
<td>(in %)</td>
<td>0.40</td>
<td>0.92</td>
<td>0.31</td>
<td>0.50</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>(in %)</td>
<td>5.68</td>
<td>0.35</td>
<td>0.83</td>
<td>2.10</td>
</tr>
<tr>
<td>$\delta$</td>
<td>(in %)</td>
<td>13.23</td>
<td>13.23</td>
<td>6.99</td>
<td>8.51</td>
</tr>
<tr>
<td>$\theta$</td>
<td>(in %)</td>
<td></td>
<td></td>
<td>2</td>
<td>*</td>
</tr>
<tr>
<td>$K_{1992}$</td>
<td>million 2004 Euro</td>
<td>24,365</td>
<td>35,411</td>
<td>211,437</td>
<td>271,213</td>
</tr>
<tr>
<td>2001 capital stock</td>
<td>million Euro</td>
<td>31,100</td>
<td>59,571</td>
<td>272,582</td>
<td>363,253</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>363,253</td>
<td></td>
</tr>
</tbody>
</table>

* Set equal for small enterprises, medium-sized enterprises and large enterprises.
Source: Panteia/EIM.

Fixed labour

According to the estimation results, a small enterprise on average has had fixed labour of 1.6 labour years equivalent to 60% of total employment. At the other extreme, a large enterprise – which on average employed 414 labour years – has a fixed labour share of 1%, equivalent to less than 3 labour years. If a small enterprise grows, the share of fixed labour initially decreases steeply. For larger enterprises, the total amount of fixed labour changes only slightly with enterprise size. See Figure 1.

The bootstrap exercise shows a clear negative correlation between the $\lambda$'s of individual size-classes, which is to be expected as their weighted average should equal the imposed aggregate value.
Figure 1 estimated relation between fixed labour (lf; lf/l) and enterprise size (l, on x-axis)

<table>
<thead>
<tr>
<th>average enterprise size, 2009</th>
<th>fixed labour</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>labour yrs.</td>
</tr>
<tr>
<td>small enterprises</td>
<td>2</td>
</tr>
<tr>
<td>medium-sized enterprises</td>
<td>26</td>
</tr>
<tr>
<td>SMEs</td>
<td>4</td>
</tr>
<tr>
<td>large enterprises</td>
<td>370</td>
</tr>
<tr>
<td>all enterprises</td>
<td>7</td>
</tr>
</tbody>
</table>

Source: Panteia/EIM; equation (18): \( l_f = \alpha_1 + \alpha_2 \cdot \ln(l) \), and \( \alpha_1 \) and \( \alpha_2 \) from Table 4.

**Technical progress**

As in the initial estimate, the estimated parameter for labour augmenting technical progress, \( \lambda \), is much stronger in small enterprises (5.7%) than in medium-sized (0.4%) and large enterprises (0.8%). However, labour augmenting technical progress affects variable labour only. So the impact of the high value for small enterprises is mitigated by the low share of variable labour in total employment, and size-class differences regarding the impact of technical progress on labour productivity growth are less than the \( \lambda \)-estimates suggest.

**Capital/labour substitution**

Variable labour and capital are easily substituted in small and medium-sized enterprises (\( \sigma \) very close to one), and much less so in large enterprises (\( \sigma = 0.3 \)). In fact the production function for SMEs could be considered as Cobb-Douglas. Still, for small enterprises the overall substitution elasticity (\( \sigma_{tot} \)) is much smaller than unity due to the existence of fixed labour, which does not react to changes in relative labour costs. It turns out that only medium-sized enterprises have an overall substitution elasticity larger than average. Both in small and large enterprises the overall substitution elasticity is less than average.
4.3 Capacity/equilibrium versus actual values

The interpretation of the production function (11) actually is what enterprises would be able to produce under 'normal' circumstances: this could be labelled 'production capacity'. Actual production is of course determined by demand for goods and services. The estimation procedure with respect to the production function thus implicitly assumes that enterprises have installed an amount of production capacity that is (on average) close to actual production. In the short run, however, differences between production capacity and actual production are to be expected.

Similarly, labour demand (equation (14)) is the amount of labour enterprises would like to employ given the amount of capital available, and in view of relative labour costs. Here too, it is assumed that on average, labour demand is close to actual employment, but in the short run, differences will exist, for instance as a result of adjustment costs. One should however also expect a tendency of actual employment to adjust to labour demand.

Analogously, it is likely that in the short run observed prices differ from their equilibrium levels, but these differences should be temporary: a tendency of actual prices to adjust to equilibrium levels should be expected.

4.3.1 Production capacity and actual production

Figure 2 shows capacity and actual production for elementary aggregates (small, medium-sized and large enterprises) as well as two aggregate categories: SMEs and all enterprises. Regarding the total private enterprise sector, developments 1993/2009 are characterised by diminishing excess capacity between 1993 and 2000. Between 2000 and 2003, actual production and production capacity roughly are in equilibrium, while actual production increasingly exceeds capacity from 2005 onwards. This development is reversed in 2009 Great Recession, when actual production drops significantly (by -5.7%) with capacity showing only a minor decrease.

Similar trends can be observed for large enterprises (which account for over half of aggregate production). Actual production in SMEs developed only modest or negative (small enterprises) in 2008 already, and capacity utilisation decreased in both 2009 and 2009. During 1993/2007, SMEs' actual production has been much closer to capacity than large enterprises'.


4.3.2 Equilibrium labour demand and actual employment

Equilibrium labour demand (i.e., the simulated values of equation (16)) and actual employment are depicted in Figure 3. The general picture is one of underemployment in the early part of the simulation period 1993/2009, and overemployment afterwards. This holds for small, mediumsized as well as large enterprises.

It can also be seen that in the (very) long run there is a tendency for actual employment to converge to equilibrium levels. This is suggested by the negative correlation between employment growth and the lagged ratio of actual employment to equilibrium labour demand. If employment is above (below) its equilibrium level, this would have a negative (positive) impact on next year’s actual employment growth. This simple relation is seen to hold for small and large enterprises1.

1 An in-depth analysis of the short run employment dynamics is beyond the scope of the this study.
4.3.3 *Equilibrium and actual price*

At the aggregate level prices have been rather close to equilibrium levels (Figure 4). However there are significant differences between SMEs on the one hand and large enterprises on the other. SMEs' prices have been below equilibrium until the turn of the century, and higher afterwards; the opposite occurred for large enterprises. The ratio between actual and equilibrium prices exhibits a higher variance for small enterprises than for medium-sized and large ones.

In medium-sized and large enterprises, there is a tendency for the actual price to return to the equilibrium level. This follows from regressing observed price change to the lagged ratio of actual to equilibrium price, which shows a negative reaction coefficient: if the actual price is above (below) equilibrium, this has a negative (positive) impact on observed price change. Such a correlation, however, has not been found for small enterprises¹.

¹ An in-depth analysis of the short run price dynamics is beyond the scope of the current study.
Figure 4  equilibrium and actual value added deflator

Source: Panteia/EIM.
5 Productivity differentials between small, medium-sized and large enterprises

5.1 Differences 2009

In 2009, labour productivity in SMEs amounted to 54,000 € per labour year, which is equivalent to 75% of average labour productivity in the private enterprise sector. Labour productivity in large enterprises was 37% above the private sector average of 71,000 € per labour year. Following the analysis here, two factors affect productivity differences: the shape of the production function, and as a result of this and the circumstances under which SMEs and large enterprises operate, differences in capital use.

Shape of the production function

As Chapter 4 clearly shows, the shape of the production function differs across enterprises size-classes. This refers to the various parameters of the production function as described in equations (5) and (11): amount of fixed labour, distribution parameter, scaling constant, state of technology, and substitution elasticity. As a result of this, a direct comparison between SMEs and large enterprises regarding productivity of one production factor does not make sense: enterprises face a trade-off between using capital and labour and seek the optimal capital/labour ratio such that marginal costs of these factors are identical\(^1\). If marginal labour productivity is relatively high (compared to relative labour costs) in comparison to the marginal productivity of capital, enterprises will use relatively much labour. This negatively affects labour productivity but it is favourable for profitability. Apart from the factor prices, labour productivity therefore also depends on the shape of the production function. Figure 5 compares actual labour productivity and the counterfactual values across small, medium-sized and large enterprises. The counterfactual labour productivity has been calculated assuming the capital/labour ratio is the same in all size-classes, equal to the average capital labour ratio across all size-classes. This could be considered as a labour productivity measure that has been adjusted for shape differences of the production function.

\(^1\) This has been used to derive the labour demand equations (13) and (16).
Figure 5  labour productivity under alternative assumptions on capital/labour ratio by enterprise size-class, private enterprise, 2009 (index, all enterprises =100)

![Bar chart showing labour productivity under different capital/labour ratios for small, medium-sized, and large enterprises.]

<table>
<thead>
<tr>
<th></th>
<th>small enterprises</th>
<th>medium-sized enterprises</th>
<th>large enterprises</th>
</tr>
</thead>
<tbody>
<tr>
<td>actual capital/labour ratio</td>
<td>58</td>
<td>91</td>
<td>137</td>
</tr>
<tr>
<td>average capital/labour ratio*</td>
<td>114</td>
<td>114</td>
<td>71</td>
</tr>
</tbody>
</table>

* Comparison across enterprise size-classes adjusts for differences in the production function parameters.
Source: Panteia/EIM.

From Figure 5 it clearly follows that if small, medium-sized and large enterprises would use the same capital/labour ratio, labour productivity in SMEs would be significantly higher than in large enterprises. Conversely, this means that SMEs’ relatively low labour productivity is actually a voluntary choice from cost-minimising enterprises, that take into account their production function as well as relative factor prices.

**Capital use**

Figure 6 shows differences in labour productivity and the capital/output ratio across small, medium-sized and large enterprises. The data clearly show labour productivity in large enterprises to benefit from a more capital-intensive production process: both small and medium-sized enterprises have below average labour productivity and capital/output ratio, whereas these ratios are above average in large enterprises. These differences in labour productivity are the result of (optimal) entrepreneurial choices taking into account within the boundaries set by their production function and (relative) factor costs enterprises face.
Labour productivity: production/total employment.
Capital/output ratio: capital stock/production.
Source: Panteia/EIM.

5.2 Developments 1993/2009

Methodology
As indicated in Chapter 1, labour productivity growth has been positively correlated with enterprise size. Using the model developed, labour productivity growth can be ascribed to two sets of factors: structural ones and temporary ones. Structural factors are labour augmenting technical progress, capital/labour substitution, and average enterprise size (because of fixed labour). Capacity utilisation and the differences between actual and equilibrium employment are considered as temporary effects as in the long run capacity will fully adjust to demand for goods and services (actual production), and actual employment adjusts to labour demand.

Because of the non-linearity of the model, the impact of these factors individually cannot be singled out analytically; instead simulation is used. A first simulation completely replicates the 1993 situation over the 1993/2009 period; subsequently all determining factors take on their actual values. Thus, in a second simulation, technical progress (\( \lambda \)), is set to its estimated value (Table 4); comparing the second with the first simulation shows the impact of technical progress on labour productivity. Then, differences between labour costs and the user costs of capital are set to their actual levels in stead of being kept constant. Comparing the results of this third simulation with the second reveals the impact of capital/labour substitution. Similarly, the impact of changes in average

---

1 Average enterprise size (in terms of employment) has been steadily decreasing over the period under investigation. For this reason decreasing average enterprise size is considered as a structural factor.
enterprise size is calculated by setting average enterprise size to its actual level in stead of its 1993 value. What remains then is the impact of temporary factors: capacity utilisation and labour hoarding\(^1\). The impact of these factors is analysed in the same way as the impact of changes in enterprise size.

**Labour productivity growth 1993/2009**

Simulation results are in Table 5. On average, labour productivity increased by 2% in the private enterprise sector. For large enterprises, the figure was significantly higher; SMEs lagged behind. For the medium term, structural factors are particularly relevant, especially because neither 1993 nor 2009 are neutral with respect to the business cycle. Looking at structural factors alone labour productivity on average increased by 1.7% annually, with above-average growth rates in medium-sized enterprise and small enterprises lagging behind.

The impact of labour augmenting technical change has been highest in small enterprises, at 2% annually. Still, this is less than the estimated technical change parameter \( \lambda \) (5.7%), as \( \lambda \) only refers to variable labour, with fixed labour making up 60% of total employment. In line with the \( \lambda \) estimates, medium-sized and large enterprises lag well behind small enterprises, notwithstanding the much smaller share of fixed labour.

If labour costs increase more than the user cost of capital – which has been the case –, firms substitute labour for capital. This contributed positively to labour productivity growth in each size-class, but somewhat more in medium-sized than in small and large enterprises. This is the results of combination of factors (labour costs and substitution elasticities). Relative labour costs have increase strongest in large enterprises; this has been almost sufficient to compensate for the low overall substitution elasticity. This elasticity is highest in medium-sized enterprises. In small enterprises, the overall substitution elasticity is not high enough to cause a high impact of capital/labour substitution on productivity growth\(^2\).

Finally, average enterprise size has decreased over the years, and as a result, the share of variable labour in total employment became larger. This negatively affects labour productivity in all three size-classes. It was largest in small enterprises.

\(^1\) 'Labour hoarding' is used here as a generic term for the differences between actual and equilibrium employment.

\(^2\) The impact of relative wage changes on productivity can be approximated by multiplying the relative labour costs by the overall elasticity of substitution:
Table 5: Contribution of factors to labour productivity growth in private enterprise, 1993/2009

<table>
<thead>
<tr>
<th></th>
<th>small enterprises</th>
<th>medium-sized enterprises</th>
<th>SMEs</th>
<th>large enterprises</th>
<th>all enterprises</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>average annual change in %</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Structural factors</td>
<td>1.3</td>
<td>2.7</td>
<td>1.9</td>
<td>1.7</td>
<td>1.7</td>
</tr>
<tr>
<td>of which</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Technical progress</td>
<td>2.0</td>
<td>0.2</td>
<td>0.9</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>- Capital/labour substitution</td>
<td>1.6</td>
<td>3.0</td>
<td>2.3</td>
<td>1.7</td>
<td>2.0</td>
</tr>
<tr>
<td>- Enterprise size</td>
<td>-2.3</td>
<td>-0.4</td>
<td>-1.3</td>
<td>-0.7</td>
<td>-1.0</td>
</tr>
<tr>
<td>Temporary factors</td>
<td>-0.5</td>
<td>-1.7</td>
<td>-1.0</td>
<td>1.5</td>
<td>0.1</td>
</tr>
<tr>
<td>of which</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Capacity utilisation</td>
<td>-0.4</td>
<td>0.3</td>
<td>0.0</td>
<td>0.6</td>
<td>0.3</td>
</tr>
<tr>
<td>- Labour hoarding</td>
<td>-0.1</td>
<td>-2.0</td>
<td>-1.0</td>
<td>0.8</td>
<td>-0.2</td>
</tr>
<tr>
<td><strong>Total labour productivity growth</strong></td>
<td>0.8</td>
<td>1.0</td>
<td>1.0</td>
<td>3.2</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Source: Panteia/EIM.

What happened in 2009?

In 2009 the Great Recession took place, with GDP falling by 3.5% and only an employment decrease of -1.1%, implying a macro-economic labour productivity decrease of -2.4%. In the private enterprise sector, labour productivity decreased by -3.5%. As can be seen from Table 6, the main contribution came from labour hoarding: enterprises have not adjusted their employment to worse market circumstances. This might have been caused by sluggishness in adjusting actual labour to labour demand; it could also have been the case that enterprises were expecting the production drop to be temporary and kept labour in hope for better times. Also policy measures taken at that time could have played a significant role (Hijzen and Venn, 2011). This labour hoarding effect has been strongest in medium-sized and large enterprises. Contrary to these, small enterprises have been able to decrease the gap between labour demand and actual employment, which contributed positively to productivity growth. Conversely, the decrease in enterprise size contributed negatively to labour productivity in small enterprises. The decrease in average enterprise size could have been the result of the number of enterprises not adjusting to market circumstances. Also the increase in the number of solo self-employed may have played a role (Vroonhof, Bruins and De Vries, 2010). It is also known that solo self-employed have seen the turnover decrease strongly during the Great Recession (EIM, 2011), which is consistent with decreasing labour productivity.
Table 6 contribution of factors to labour productivity growth in private enterprise, 2009

<table>
<thead>
<tr>
<th></th>
<th>small enterprises</th>
<th>medium-sized enterprises</th>
<th>SMEs</th>
<th>large enterprises</th>
<th>all enterprises</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>structural factors</td>
<td>-8.7</td>
<td>6.8</td>
<td>-1.2</td>
<td>3.7</td>
<td>1.1</td>
</tr>
<tr>
<td>of which</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- technical progress</td>
<td>2.0</td>
<td>0.2</td>
<td>1.0</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>- capital/labour subst.</td>
<td>1.6</td>
<td>3.6</td>
<td>2.5</td>
<td>2.1</td>
<td>2.2</td>
</tr>
<tr>
<td>- enterprise size</td>
<td>-11.9</td>
<td>3.0</td>
<td>-4.5</td>
<td>0.8</td>
<td>-1.8</td>
</tr>
<tr>
<td>temporary factors</td>
<td>5.8</td>
<td>-13.6</td>
<td>-4.6</td>
<td>-5.3</td>
<td>-5.1</td>
</tr>
<tr>
<td>of which</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- capacity utilisation</td>
<td>3.4</td>
<td>-3.6</td>
<td>-0.9</td>
<td>-0.9</td>
<td>-1.1</td>
</tr>
<tr>
<td>- labour hoarding</td>
<td>2.3</td>
<td>-10.3</td>
<td>-3.8</td>
<td>-4.4</td>
<td>-4.0</td>
</tr>
<tr>
<td>total labour productivity growth</td>
<td>-3.5</td>
<td>-7.7</td>
<td>-5.8</td>
<td>-1.8</td>
<td>-3.5</td>
</tr>
</tbody>
</table>

Source: Panteia/EIM.

An interesting discussion might be whether the impact of decreasing enterprise size in particular in small enterprises should be in fact considered as labour hoarding. In a model without fixed labour it is not possible to distinguish between these effects. In medium-sized and large enterprises – with a moderate impact of fixed labour – labour hoarding plays a significant role in the explanation of the 2009 labour productivity decrease, whereas a similar role is played by decreasing enterprise size in small enterprises. However, the decrease in enterprise size observed in small (and other) enterprises in 2009 does not deviate significantly from the trend starting in 2002 and its impact on labour productivity should therefore not be interpreted as a phenomenon directly related to the Great Recession.
Figure 7  average enterprise size by enterprise size-class, private enterprise, 1993-2009

<table>
<thead>
<tr>
<th>Year/Enterprise</th>
<th>1993</th>
<th>2004</th>
<th>2009</th>
</tr>
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<tbody>
<tr>
<td>small enterprises</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>medium-sized enterprises</td>
<td>29</td>
<td>28</td>
<td>26</td>
</tr>
<tr>
<td>large enterprises</td>
<td>426</td>
<td>416</td>
<td>370</td>
</tr>
</tbody>
</table>

Source: Panteia/EIM.
6 Summing up

Labour productivity differs significantly between small, medium-sized and large enterprises in the private enterprise sector. This holds both for the level of labour productivity as well as its development. As has been shown elsewhere (Kwaak, 2011), size-class differences regarding the level of labour productivity are only to a limited extent dependent on differences in sectoral structure. A model of the production structure has been developed and estimated to shed some light on these phenomena.

An important property is the introduction of fixed cost. It turns out (in the estimation results) that fixed costs are such that economies of scale exist.

Labour productivity in 2009
According to the model, labour productivity differentials can be explained by the following factors:
- The shape of the production function. This refers in particular to the existence of fixed labour, and the marginal productivity of labour and capital varying across size-classes.
- Differences in capital use. As a result of differences in the shape of the production function as well as differences in labour costs relative to capital costs across size-classes, the capital intensity of production is much higher in large enterprises compared to SMEs which contributes to higher labour productivity in large enterprises.

If labour productivity were calculated assuming the same capital/labour ratio across all size-classes – which is not optimal! – then labour productivity in SMEs would be higher than in large enterprises. So observed differences in labour productivity are the result of differences in the shape of the production function and – within the conditions set by the production function – of entrepreneurial choices. From this point of view the relatively low labour productivity of SMEs is no issue for direct concern.

Labour productivity growth
Regarding differences in labour productivity growth in small, medium-sized and large enterprises, the following factors have played a major role in the structural development labour productivity between 1993 and 2009:
- Labour-augmenting technical progress. In conformity with other studies, technical progress is assumed to be labour-augmenting. It only affects productivity of variable labour. The impact of labour-augmenting technical progress is by far highest in small enterprises, and below average in medium-sized and large enterprises.
- Capital/labour substitution. Variable labour is substituted by capital if labour costs increase more than capital costs, and vice versa. This effect has been rather weak in medium-sized enterprises, and slightly above average in small and large enterprises.
- Changes in the share of fixed labour. This has had a negative impact on labour productivity growth in all size-classes, but mostly so in small enterprises because of the high share of fixed labour in this size-class.
The 2009 Great Recession has negatively affected labour productivity in the private enterprise sector. This has mainly been the result of decreasing capacity utilisation, and of labour hoarding. The latter only occurred in medium-sized and large enterprises; small enterprises in fact closed the gap between labour demand and actual employment. On the other hand, due to shrinking enterprise size, the share of fixed labour in total employment in small enterprises increased significantly, which in turn contributed negatively to labour productivity growth in small enterprises. This is, however, not strictly related to the Great Recession, but fits well in the long-term trend of decreasing enterprise size.

Further research

The research described here might be extended in three directions. First, it may be applied to individual economic sectors. In particular, it seems worthwhile to review whether differences between size-classes found for private enterprise also hold within individual economic sectors. Specific parameters of interest are size-class differences in the capital/labour ratio, the role of fixed labour, the rate of technical progress, and the substitution between capital and labour. This topic is dealt with in a separate paper (Kwaak, 2012).

Secondly, following the assumption of full competition in markets, the model from Chapter 2 has implications for the number of enterprises. Prices are set such that there are no abnormal profits. However, with the existence of fixed costs, this can only occur at a specific enterprise size, i.e. if the size of the enterprise is such that fixed costs are exactly covered by the mark-up of prices over variable costs (compare equation (17)): the condition

\[ Y \cdot p_y = K \cdot p_k + L \cdot pl + N \cdot I \cdot pl \]

should hold. From this it follows that the equilibrium number of enterprises equals

\[ N^* = \frac{Y \cdot p_y - (K \cdot p_k + L \cdot pl)}{I \cdot pl} \]

The consequences of this corollary should be reviewed.

Thirdly, the short run development of both employment and prices may be analysed in view of the hypothesis that there is a long run tendency for actual employment and actual prices to move towards equilibrium levels (similar to an Error Correction framework). This would mean a stronger (econometric) evidence for the model developed here.
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ANNEX I  Formal description of the estimator

\[
\begin{align*}
\text{maximise} & \quad \beta_s, \alpha_s, \rho_s, \delta_s, \theta_s, \\
\lambda_s, \varphi_{1,s}, \varphi_{2,s}, K_{t0,s} & \quad \Lambda = \text{PSR}_2 + \text{PSR}_2 + \text{PSR}_2 + \text{PSR}_{py}
\end{align*}
\]

where \(\text{PSR}_2 = 1 - \frac{(\sum (z - \hat{z})^2/n)}{\text{var}(z)}\),

with \(z\) the observed, and \(\hat{z}\) the estimated value of variable \(Z\), and \(Z \in \) production (\(Y\)), depreciation (\(D\)), total employment (\(L\)), price index (\(py\))

Under the following constraints across size-classes (with \(w_s\) an appropriate weight for size-class \(s\)):

\[(I.1) \quad \sum S w_s \cdot \sigma_{tot,s} = \sigma_{CPB} \quad \text{consistency with industry substitution elasticity. Note that } \sigma_{tot} = (L_f/L) \sigma, \text{ and } \sigma = 1/(1+\rho)\]

\[(I.2) \quad \sum S w_s \cdot \lambda_s = \lambda_{CPB} \quad \text{consistency with industry technical progress}\]

\[(I.3) \quad \sum S K_{2001,s} = K_{2001,CBS} \quad \text{consistency with industry capital stock}\]

\[(I.4) \quad \sum S N_s \cdot L_{s}/(L_f/L)_{CPB} \quad \text{consistency with industry share of fixed labour}\]

Also non-negativity constraints are imposed. Additionally, to avoid stability problems, the constraint \(\rho_s > 0.001\) has been imposed\(^1\)

Note that

- \(K_{t0}\) is the stock of capital in the year prior to the estimation period.
- capital stock changes as a result of gross investment and depreciation (as in the model above), but also because of sales of second hand capital goods and re-valuations. Therefore only consistency with aggregate capital stock for one year is imposed in (I.3).
- The selected model from section 4.2 does not include a size-class index for \(\gamma_1\) and \(\gamma_2\) as these parameters are estimated across size-classes.

\(^{1}\) With \(\rho = 0\), the production function collapses into a Cobb-Douglas function \(Y = \beta \cdot K^\alpha \cdot L^{(1-\alpha)}\).
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